CHAPTER – 3

PAIR OF LINEAR EQUATION IN TWO VARIABLES

(1) An equation in the form $ax + by + c = 0$, where a, b and c are real numbers, and $a \neq 0$ and $b \neq 0$, is called a linear equation in two variables x and y.

***For Example*:** $2x + 3y + 7 = 0$, where $a = 2$, $b = 3$, $c = 5$ are real numbers. So, given equation is a linear equation in two variables.

(2) Each solution (x, y) of a linear equation in two variables, $ax + by + c = 0$, corresponds to a point on the line representing the equation, and vice versa.

***For Example*:** $2x + 3y = 5$ has (1, 1) as its solution. So,(1, 1) will lie on the line $2x + 3y = 5$.

DO AND LEARN

Find three solutions of equation $2x + 3y = 5$ and draw graph of it.

What is the shape of the graph?

How many points lie on the graph of the line?

(3) The general form for a pair of linear equations in two variables x and y is :

$$a_1x + b_1 y + c_1 = 0 \text{ and}$$
$$a_2x + b_2 y + c_2 = 0,$$

where a_1 , b_1 , c_1 , a_2 , b_2 , c_2 are all real numbers and $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$.

***For Example*:** $2x + 3y - 7 = 0$ and $9x - 2y + 8 = 0$ forms a pair of linear equations.

Methods of solving pair of Linear Equation in two Variables:

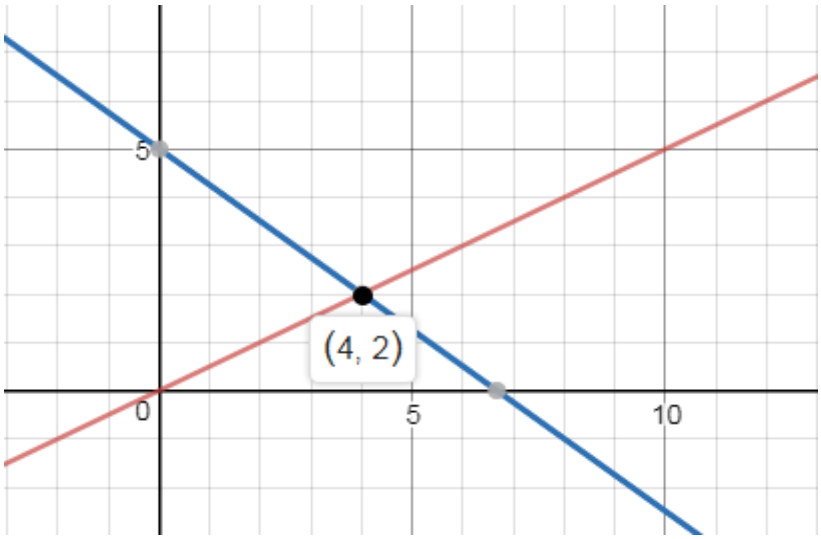
1. **Graphical Method**
2. **Substitution Method**
3. **Elimination Method**
4. **Cross Multiplication Method**

1. Graphical Method:

- **Graph of a linear equation is a straight line.**
- **Therefore graph of pair of linear equation in two variables is pair of straight lines.**
- **Now, there are three possible cases.**

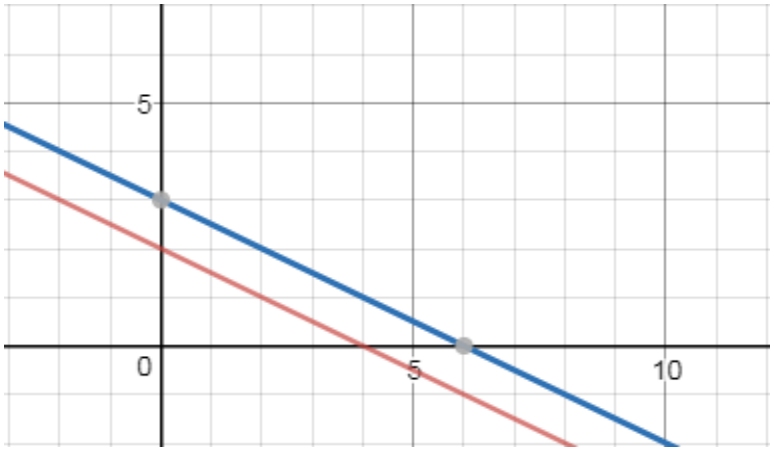
case(i) If the graphs of two equations of a system intersect at a point, the system is said to have a **unique solution,** i.e., the system is **consistent.**

***For Example*:** $x - 2y = 0$ and $3x + 4y - 20$ intersects each other at unique point (4, 2).



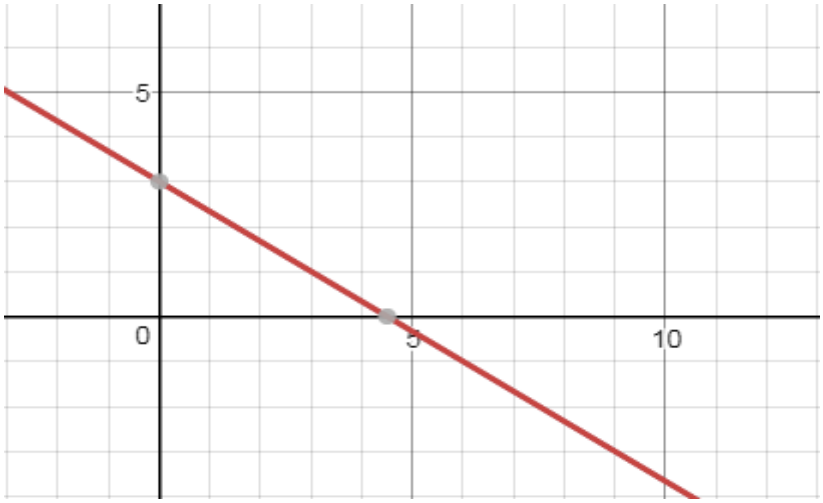
case(ii) If the graphs of two equations of a system are two parallel lines, the system is said to have **no solution,** i.e., the system is **inconsistent.**

***For Example*:** $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ are parallel lines.



case(iii) When the graphs of two equations of a system are two coincident lines, the system is said to have **infinitely** many solutions, i.e., the system is **consistent and dependent.**

***For Example*:** $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ are coincident lines.



Conclusion: If we draw the graph of both equations

$a_1x + b_1 y + c_1 = 0$ and

$a_2x + b_2 y + c_2 = 0$,

Then three possible cases are there:

1. If lines intersect at a single point, then it has a unique solution and the system is said to be consistent.
   In this case, $\frac{a1}{a2} \neq \frac{b1}{b2}$
2. If lines are parallel, then equations have no solution and the system is said to be inconsistent.
   In this case, $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$
3. If lines are coincident, then equations have infinitely many solutions and the system is said to be consistent.
   In this case, $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$

SUMMARY:

Pair of lines $a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratio
$2x+3y+4=0$ $5x+6y+9=0$	$\frac{2}{5}$	$\frac{3}{6}$	$\frac{4}{9}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$x+2y+5=0$ $3x+6y+15=0$	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{5}{15}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$2x-3y+4=0$ $4x-6y+10=0$	$\frac{2}{4}$	$\frac{-3}{-6}$	$\frac{4}{10}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Graphical representation	Algebraic interpretation
Intersecting lines	Exactly one solution (unique)
Coincident lines	Infinitely many solutions
Parallel lines	No solution

From the table above you can observe that if the line $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

(i)	for the intersecting lines then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
(ii)	for the coincident lines then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
(iii)	for the parallel lines then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

EXAMPLE: For what value of k, pair of linear equations $kx + 3y + 2 = 0$, $2x + y + 3 = 0$ has no solutions?

SOL: The given equations are:

$kx + 3y + 2 = 0$

$2x + y + 3 = 0$

Here, $a_1 = k$, $b_1 = 3$, $c_1 = 2$, $a_2 = 2$, $b_2 = 1$, $c_2 = 3$

Now, for no solution, $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$

⇨ $\frac{k}{2} = \frac{3}{1} \neq \frac{2}{3}$

⇨ $\frac{k}{2} = \frac{3}{1}$ and $\frac{k}{2} \neq \frac{2}{3}$

⇨ $k = 6$ and $k \neq \frac{4}{3}$

⇨ $k = 6$

PRACTICE PROBLEMS

1. Solve the following pair of linear equations **graphically.**
   i) $2x + y = 6$
      $4x - 2y = 4$
   ii) $2x - 2y = 2$
      $4x - 4y = 5$
   iii) $x - y + 1 = 0$
      $3x + 2y - 12 = 0$
   iv) $5x - 6y + 30 = 0$
      $5x + 4y - 20 = 0$
2. Find k for which $5x + 10y - 2 = 0$ and $10x + ky + 4 = 0$ lines are parallel to each other.
3. For what k the equations $2x + 6y - 1 = 0$ and $4x + ky - 3 = 0$ are parallel?
4. For what value of k, pair of linear equations, $2x + 3y = 7$, $4x + ky = 14$, has infinitely many solutions?

**** THIS WORKSHEET IS PREPARED AT HOME.**