DIFFERENTIATION

- **Derivative of a function :** Suppose f is a real function and c is a point in its domain. The derivative of f at c is defined by $f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ provided this limit exists.
- $\frac{dy}{dx}$ is derivative of first order and is also denoted by y' or y_1.
- Sum, difference, product and quotient of continuous functions are continuous. i.e., if f and g are continuous functions, then

  i) $(f + g) = f(x) + g(x)$ is continuous.
  ii) $(f - g) = f(x) - g(x)$ is continuous.
  iii) $(f \cdot g)(x) = f(x) \cdot g(x)$ is continuous.
  iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ (wherever $g(x) \neq 0$) is continuous.

- Every differentiable function is continuous, but the converse is not true.
- **Following are some of the standard derivatives (in appropriate domains):**

1. $(u \pm v)' = u' \pm v'$
2. $(uv)' = u'v + uv'$ **[Product Rule]**
3. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$, **wherever** $v \neq 0$ **[Quotient Rule]**
4. **If** $y = f(u); u = g(x)$, **then** $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ **[Chain Rule]**
5. **If** $x = f(t); y = g(t)$, **then** $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ **[Parametric Form]**
6. $\frac{d}{dx}(x^n) = nx^{n-1}$
7. $\frac{d}{dx}(\sin x) = \cos x$
8. $\frac{d}{dx}(\cos x) = -\sin x$
9. $\frac{d}{dx}(\tan x) = -\sec^2 x$
10. $\frac{d}{dx}(\cot x) = -\cos ec^2 x$
11. $\frac{d}{dx}(\sec x) = \sec x . \tan x$
12. $\frac{d}{dx}(\cos ecx) = -\cos ecx . \cot x.$

13. $\frac{d}{dx}(a^x) = a^x.\log_e a$

14. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

15. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

16. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

17. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

18. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{1+x^2}$

19. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{1-x^2}}$

20. $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$

21. $\frac{d}{dx}(e^x) = e^x$

PRACTICE PROBLEMS

1. Find $\frac{dy}{dx}$, if tanx – y = e^x
2. Find $\frac{d}{dx}$sin(5-3x).
3. Find $\frac{dy}{dx}$, if $x^2 + y^2 = 5$
4. Differentiate 5^{sinx} w.r.t. x.
5. Find $\frac{d^2y}{dx^2}$, if y = logx
6. Differentiate $\log x^2$ w.r.t. x.
7. Find $\frac{dy}{dx}$ if y = sin ($\tan^{-1}e^{-x}$).
8. Differentiate y = log(cos e^x) w.r.t. x
9. If y = $\sin^{-1}x$, prove that :

   $(1 - x^2)\frac{d2y}{dx2} - x\frac{dy}{dx} = 0$
10. If y = $\sin^{-1}x$, prove that :

    $(1 - x^2)\frac{d2y}{dx2} - x\frac{dy}{dx} = 0$

**** THIS WORKSHEET IS PREPARED AT HOME**