WORKSHEET ON DETERMINANTS

Properties of Minors and Cofactors

(i) The sum of the products of elements of .any row (or column) of a determinant with the cofactors of the corresponding elements of any other row (or column) is zero, i.e., if

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Then, $\mathbf{a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33} = 0}$ and so on.

DO AND LEARN

Verify this property using $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

(ii) The sum of the product of elements of any row (or column) of a determinant with the cofactors of the corresponding elements of the same row (or column) is Δ

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Then, $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

DO AND LEARN

Verify this property using $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

Adjoint of a Matrix :-

Let $A[a_{ij}]$ m x n be a square matrix of order m x n and let C_{ij} be the cofactor of a_{ij} in the determinant |A| , then the Adjoint of A, denoted by adj (A), is defined as the transpose of the matrix, formed by the cofactors of the matrix.

$$\text{adj}(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

Properties of Adjoint of a Square Matrix

If A and B are square matrices of order n, then

1. A (adj A) = (adj A) A = |A|I
2. adj (A') = (adj A)'
3. adj (AB) = (adj B) (adj A)
4. adj (kA) = k^{n-1}(adj A), k ∈ R
5. adj (A^m) = $(\text{adj } A)^m$
6. |adj A| =$|A|^{n-1}$,A is a non-singular matrix.
7. Adjoint of a diagonal matrix is a diagonal matrix.

PRACTICE PROBLEMS

1. If A is a square matrix of order 3 and $|3A| = k\,|A|$, then write the value of k.

2. Let A be a non singular square matrix of order 3 X 3. Then find the value of |adj A| .

3. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

4. Write the value of the determinant $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

5. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

6. Solve all the problems from Ex–4.5