MOTION IN A STRAIGHT LINE

Introduction
Motion is one of the significant topics in physics. Everything in the universe moves. It might only be a small amount of movement and very-very slow, but movement does happen. Even if you appear to be standing still, the Earth is moving around the sun, and the sun is moving around our galaxy.
“An object is said to be in motion if its position changes with time”.
The concept of motion is a re’ live one and a body that may be in motion relative to one reference system, may be at rest relative to another.
There are two branches in physics that examine the motion of an object.
(i) Kinematics: It describes the motion of objects, without looking at the cause of the motion.
(ii) Dynamics: It relates the motion of objects to the forces which cause them.

• Point Object
If the length covered by the objects are very large in comparison to the size of the objects, the objects are considered point objects.

• Reference Systems
The motion of a particle is always described with respect to a reference system. A reference system is made by taking an arbitrary point as origin and imagining a co-ordinate system to be attached to it. This co-ordinate system chosen for a given problem constitutes the reference system for it. We generally choose a co-ordinate system attached to the earth as the reference system for most of the problems.

• Total Path Length (Distance)
For a particle in motion the total length of the actual path traversed between initial and final positions of the particle is known as the ‘total path length’ or distance covered by it.

• Types of Motion
In order to completely describe the motion of an object, we need to specify its position. For this, we need to know the position co-ordinates. In some cases, three position co-ordinates are required, while in some cases two or one position co-ordinate is required.
Based on these, motion can be classified as:
(i) One dimensional motion. A particle moving along a straight-line or a path is said to undergo one dimensional motion. For example, motion of a train along a straight line, freely falling body under gravity etc.
(ii) Two dimensional motion. A particle moving in a plane is said to undergo two dimensional motion. For example, motion of a shell fired by a gun, carrom board coins etc.
(iii) Three dimensional motion. A particle moving in space is said to undergo three dimensional motion. For example, motion of a kite in sky, motion of aeroplane etc.

• **Displacement**
Displacement of a particle in a given time is defined as the change in the position of particle in a particular direction during that time. It is given by a vector drawn from its initial position to its final position.
• **Factors Distinguishing Displacement from Distance**
—> Displacement has direction. Distance does not have direction.
—> The magnitude of displacement can be both positive and negative.
—> Distance is always positive. It never decreases with time.
—> Distance ≥ | Displacement |

Distance	*Displacement*
(*i*) Length of actual path covered between the initial and final positions/points	(*i*) Length of the shortest path between initial and final points.
(*ii*) Scalar quantity	(*ii*) Vector quantity
(*iii*) Can have only +ve values	(*iii*) Can have –ve, 0, +ve values.

→ Both distance and displacement are measured in metres or kilometre. Their dimension is [L].

Speed	*Velocity*
(*i*) The rate at which distance is covered.	(*i*) The rate at which displacement takes place.
(*ii*) Scalar quantity	(*ii*) Vector quanting.
(*iii*) Can have only +ve values.	(*iii*) Can have –ve, 0, +ve values.

Speed and velocity are expressed in metre per sec, *i.e.*, ms^{-1}. The dimensional formula is $[LT^{-1}]$.

$$\text{Velocity} = \frac{\text{Final position} - \text{Initial position}}{\text{time}} = \frac{x_f - x_i}{t}$$

Velocity can change either by altering magnitude or by changing direction or both.

• **Uniform Speed and Uniform Velocity**
Uniform Speed. An object is said to move with uniform speed if it covers equal distances in equal intervals of time, howsoever small these intervals of time may be.
Uniform Velocity. An object is said to move with uniform velocity if it covers equal displacements in equal intervals of time, howsoever small these intervals of time may be.

• **Variable Speed and Variable Velocity**
Variable Speed. An object is said to move with variable speed if it covers unequal distances in equal intervals of time, howsoever small these intervals of time may be.
Variable Velocity. An object is said to move with variable velocity if it covers unequal displacements in equal intervals of time, howsoever small these intervals of time may be.

• **Average Speed and Average Velocity**

Average Speed. It is the ratio of total path length traversed and the corresponding time interval.

Or

"The distance covered in unit time is called average speed".

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$V_{av} = \frac{\Delta x}{\Delta t}$$

Average Velocity. Average velocity is the displacement divided by the time interval in which the displacement occurs.

Or

"It is that single velocity with which the object can travel the same length in the same time as it generally does with varying velocity".

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

$$\boxed{\vec{V}_{av} = \frac{\Delta \vec{x}}{\Delta t}}$$

The average speed of an object is greater than or equal to the magnitude of the average velocity over a given time interval.

• **Instantaneous Speed and Instantaneous Velocity**

Instantaneous Speed. The speed of an object at an instant of time is called instantaneous speed.

Or

"Instantaneous speed is the limit of the average speed as the time interval becomes infinitesimally small".

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta \bar{x}}{\Delta t} = \frac{dx}{dt}}$$

Instantaneous velocity

The instantaneous velocity of a particle is the velocity at any instant of time or at any point of its path.

or

"Instantaneous velocity or simply velocity is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small."

$$\boxed{\bar{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}}$$

• **Acceleration**

The rate at which velocity changes is called acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$\boxed{a = \frac{v-u}{t}}$$

where v and u are final and initial velocity respectively. It is a vector quantity with *S.I.* unit of m/s^2 and has dimensions of $[LT^{-2}]$.

If acceleration is −*ve* (negative), then it is called retardation or deceleration.

• **Uniform Acceleration**

If an object undergoes equal changes in velocity in equal time intervals it is called uniform acceleration.

• Average and Instantaneous Acceleration

Average Accelerating. It is the change in the velocity divided by the time-interval during which the change occurs.

$$\boxed{\vec{a} = \frac{\Delta\vec{v}}{\Delta t}}$$

Instantaneous Acceleration. It is defined as the limit of the average acceleration as the time-interval Δt goes to zero.

$$\boxed{\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{dv}{dt}}$$

• **Kinematical Graphs**

The 'displacement-time' and the 'velocity-time' graphs of a particle are often used to provide us with a visual representation of the motion of a particle. The 'shape' of the graphs depends on the initial 'co-ordinates' and the 'nature' of the acceleration of the particle (Fig.)

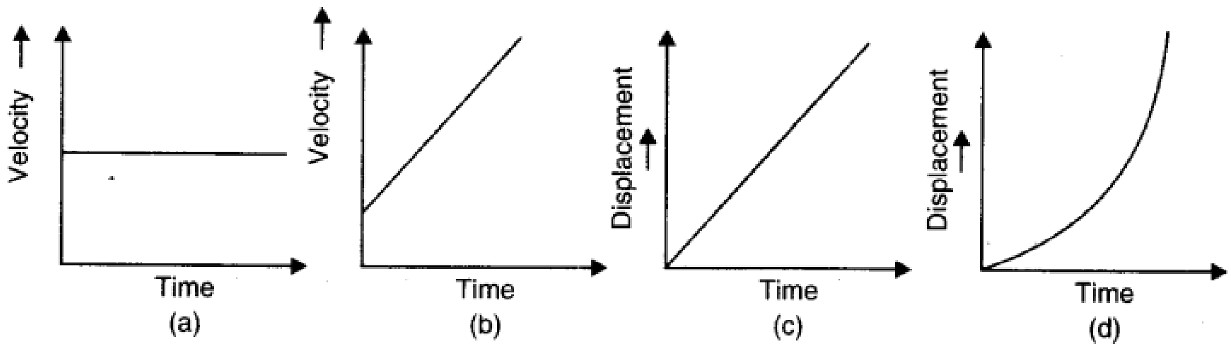


Fig. *Curves (a) and (c) represent motion with a constant speed u. Curves (b) and (d) represent motion with a uniform acceleration* ***a*** *starting with an initial speed* ***u****.*

The following general results are always valid

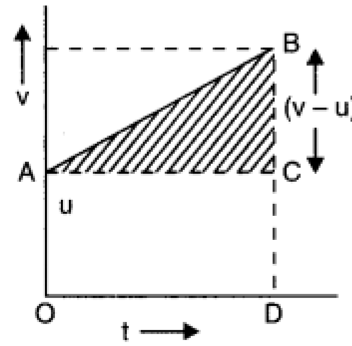
(i) The slope of the displacement-time graph at any instant gives the speed of the particle at that instant.

(ii) The slope of the velocity-time graph at any instant gives the magnitude of the acceleration of the particle at that instant.

(iii) The area enclosed by the velocity-time graph, the time-axis and the two co-ordinates at ,time instants t_1 to t_2 gives the distance moved by the particle in the time-interval from t_1 to t_2.

• Equations of Motion for Uniformly Accelerated Motion

For uniformly accelerated motion, some simple equations can be derived that relate displacement (x), time taken (f), initial velocity (u), final velocity (v) and acceleration (a). Following equation gives a relation between final and initial velocities v and u of an object moving with uniform acceleration a: $v = u + at$



This relation can be graphically represented by following figure:

The area under this curve is:

Area between instants 0 and t

$$= \text{Area of triangle } ABC + \text{Area of rectangle } OACD$$

$$= \frac{1}{2}(v-u)t + ut$$

The area under $v - t$ curve represents the displacement. Therefore, the displacement x of the object is:

$$x = \frac{1}{2}(v-u)t + ut$$

But $\quad v - u = at$

So, $\quad x = \frac{1}{2}at^2 + ut$

or, $\quad x = ut + \frac{1}{2}at^2$

The equation for displacement can also be given as follows:

$$x = \frac{v+u}{2}t = vt$$

Earlier we have derived:

$$v = u + at$$

or, $\quad \frac{v+u}{a} = t$

Substituting the value of t in equation for displacement we get,

$$x = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{2}\right)$$

or, $\quad x = \frac{v^2 - u^2}{2a}$

or, $\quad v^2 = u^2 + 2ax$

So, we have derived following kinematic equation.

$$v = u + at$$

$$x = ut + \frac{1}{2}at^2$$

$$v = u^2 + 2ax$$

If acceleration is uniform or constant, then equations of motion are:

$$v = u + at \quad ...(i)$$

$$s = ut + \frac{1}{2}at^2 \quad ...(ii)$$

$$v^2 = u^2 + 2as \quad ...(iii)$$

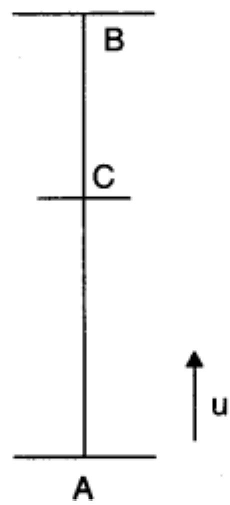
where u is initial velocity, v is final velocity, a is acceleration and s is the distance covered in time interval t.

For uniformly accelerated motion along a straight line, displacement in a particular instant of time (n^{th} second of the motion) is given by

$$s_{nth} = u + \frac{1}{2}a\,(2n - 1)$$

• Suppose a body is projected vertically upward from a point A with velocity u.

If we take upward direction as positive



(i) At time t, its velocity $v = u - gt$

(ii) At time t, its displacement from A is given by

$$h = ut - \frac{1}{2}gt^2$$

(iii) Its velocity when it has a displacement 'h' is given by

$$v^2 = u^2 - 2gh$$

(iv) When it reaches the maximum height from A, its velocity $v = 0$. This happen when $t = \frac{u}{g}$. The body is instantaneously at rest at the highest points.

(v) The maximum height reached

$$H = \frac{u^2}{2g}$$

(vi) Total time to go up and return to the point of projection $= \frac{2u}{g}$.

(vii) At any point C between A and B, where $AC = S$, the velocity v is given by

$$v = \pm\sqrt{u^2 - 2gs}$$

The velocity of body while crossing C upwards $= +\sqrt{u^2 - 2gs}$ and while crossing C downwards is $-\sqrt{u^2 - 2gs}$.

In some problems it is convenient to take the downward direction as positive, in such case all the measurements in downward direction are considered as positive i.e., acceleration will be +g. But sometimes we may need to take upward as positive and if such case acceleration will be -g.

• **Relative Velocity**

Relative velocity of an object A with respect to another object B is the time rate at which the object A changes its position with respect to the object B.

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$, where $\vec{V}_A$ and $\vec{V}_B$ are the velocities of object A and B. $\left(\vec{V}_A - \vec{V}_B\right)$ indicates the addition of negative of velocity of B to the velocity of A.

—> The relative velocity of two objects moving in the same direction is the difference of the speeds of the objects.

—> The relative velocity of two objects moving in opposite direction is the sum of the speeds of the objects.

• IMPORTANT TABLES

TABLE 3.1 Some physical quantities, symbols, dimensions and their units.

S.No.	Physical quantity	Symbol	Dimensions	Units
(*i*)	Path length		[L]	m
(*ii*)	Displacement	Δx	[L]	m
(*iii*)	Velocity		$[LT^{-1}]$	ms^{-1}
	(*a*) Average	$\bar{v}$		
	(*b*) Instantaneous	v		
(*iv*)	Speed		LT^{-1}	ms^{-1}
	(*a*) Average			
	(*b*) Instantaneous			
(*v*)	Acceleration		$[LT^{-2}]$	ms^{-2}
	(*a*) Average	$\bar{a}$		
	(*b*) Instantaneous	a		

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