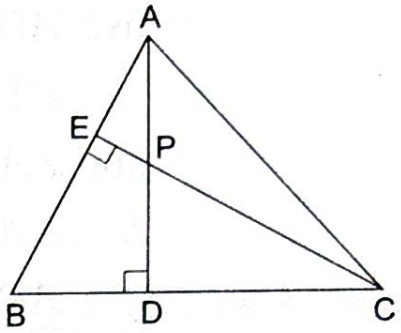
EXAMPLE 9 *In the given figure, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P.*



Show that:

(i) $\triangle AEP \sim \triangle CDP$

(ii) $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$

(iv) $\triangle PDC \sim \triangle BEC$

SOLUTION

(i) In $\triangle AEP$ and $\triangle CDP$, we have:

$\angle AEP = \angle CDP$ [each equal to 90°]

$\angle APE = \angle CPD$ [vertically opposite ∠s]

$\therefore$ $\triangle AEP \sim \triangle CDP$ [by AA-similarity].

(ii) In $\triangle ABD$ and $\triangle CBE$, we have:

$\angle ADB = \angle CEB = 90°$

$\angle B = \angle B$ [common]

$\therefore$ $\triangle ABD \sim \triangle CBE$ [by AA-similarity].

(iii) In $\triangle AEP$ and $\triangle ADB$, we have:

$\angle AEP = \angle ADB = 90°$

$\angle EAP = \angle BAD$ (common)

Hence, $\triangle AEP \sim \triangle ADB$ [by AA-similarity].

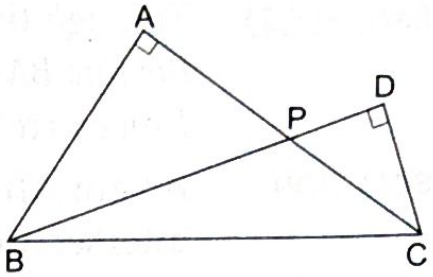
(iv) In $\triangle PDC$ and $\triangle BEC$, we have:

$\angle PDC = \angle BEC = 90°$

$\angle PCD = \angle ECB$ (common)

$\therefore$ $\triangle PDC \sim \triangle BEC$ [by AA-similarity].

EXAMPLE 19 *Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that* $AP \times PC = BP \times PD$.



[CBSE 2000C]

SOLUTION

GIVEN ΔABC and ΔDBC are drawn on the same hypotenuse BC and on the same side of BC. Also, AC and BD intersect at P.

TO PROVE $AP \times PC = BP \times PD$.

PROOF In ΔBAP and ΔCDP, we have:

$\angle BAP = \angle CPD = 90°$

$\angle BPA = \angle CPD$ (ver. opp. $\angle$s)

$\therefore \quad \Delta BAP \sim \Delta CDP$ [by AA-similarity]

$\therefore \quad \dfrac{AP}{DP} = \dfrac{BP}{CP}$

$\Rightarrow \quad AP \times CP = BP \times DP$

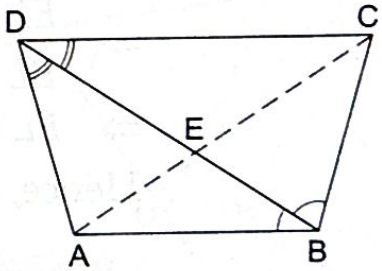
$\Rightarrow \quad AP \times PC = BP \times PD$

Hence, $AP \times PC = BP \times PD$.

EXAMPLE 20 *If the diagonal BD of a quadrilateral ABCD bisects both* $\angle B$ *and* $\angle D$, *prove that* $\dfrac{AB}{BC} = \dfrac{AD}{CD}$.

SOLUTION

GIVEN A quad. $ABCD$ in which diagonal BD bisects both $\angle B$ and $\angle D$.



TO PROVE $\dfrac{AB}{BC} = \dfrac{AD}{CD}$.

CONSTRUCTION Join AC, intersecting BD at E.

PROOF In ΔCBA, BE is the bisector of $\angle ABC$.

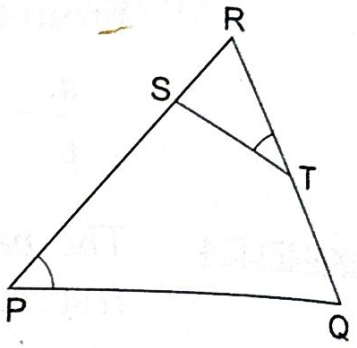
$\therefore \quad \dfrac{AE}{EC} = \dfrac{AB}{BC}$ ... (i) [by the angle-bisector theorem]

In ΔADC, DE is the bisector of $\angle ADC$.

$\therefore \quad \dfrac{AE}{EC} = \dfrac{AD}{CD}$ ... (ii) [by the angle-bisector theorem]

From (i) and (ii), we get: $\dfrac{AB}{BC} = \dfrac{AD}{CD}$.

EXAMPLE 7 *In the given figure, S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.*



SOLUTION GIVEN ΔRPQ and ΔRTS in which $\angle P = \angle RTS$.

TO PROVE $\Delta RPQ \sim \Delta RTS$.

PROOF In ΔRPQ and ΔRTS, we have:

$\angle P = \angle RTS$ (given)

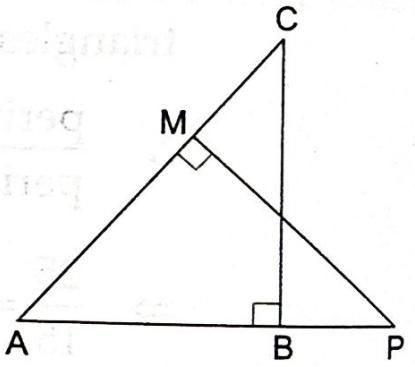
$\angle R = \angle R$ (common)

$\therefore$ $\Delta RPQ \sim \Delta RTS$ [by AA-similarity].

EXAMPLE 8 *In the given figure, ΔABC and ΔAMP are right-angled at B and M respectively.*

Prove that: (i) $\Delta ABC \sim \Delta AMP$

$$(ii)\ \frac{CA}{PA} = \frac{BC}{MP}$$



SOLUTION GIVEN ΔABC and ΔAMP such that $\angle B = 90°$ and $\angle M = 90°$.

TO PROVE (i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$.

PROOF In (i) ΔABC and ΔAMP, we have:

$\angle ABC = \angle AMP = 90°$

$\angle A = \angle A$ (common)

$\therefore$ $\Delta ABC \sim \Delta AMP$ [by AA-similarity].

(ii) Since $\Delta ABC \sim \Delta AMP$, their corresponding sides are proportional.

$\therefore$ $\frac{CA}{PA} = \frac{BC}{MP}$.