Irrational Numbers

An **Irrational Number** is a real number that **cannot** be written as a simple fraction.

(Irrational means **not Rational)**

1.5 = $\frac{3}{2}$ **(**Ratio) π = 3.14159…….= $\frac{?}{?}$ (No ratio)

(Rational number) (Irrational number)

Definition of Rational and Irrational Numbers

A **Rational** Number **can** be written as a **Ratio** of two integers (ie a simple fraction).

Example: **1.5** is rational, because it can be written as the ratio **3/2**

Example: **7** is rational, because it can be written as the ratio **7/1**

Example: **0.333...** (3 repeating) is also rational, because it can be written as the ratio **1/3**

Irrational Numbers

But some numbers **cannot** be written as a ratio of two integers they are called **Irrational Numbers**.

Example: **π (Pi)** is a famous irrational number.

π = 3.1415926……….... (and more)

We **cannot** write down a simple fraction that equals Pi.

The popular approximation of $^{22}/_{7}$ = 3.1428571428571... is close but **not accurate**.

Another point is that the decimal goes on forever without repeating cannot be written as a fraction

It is **irrational** because it cannot be written as a **ratio** (or fraction),

So we can tell if it is Rational or Irrational by trying to write the number as a simple fraction.

Example: **9.5** can be written as a simple fraction like this:

9.5 = 19/2

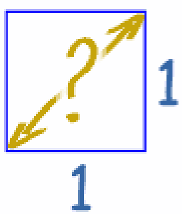
So it is a **rational number** (and so is **not irrational**)

Here are some more examples:

Number	As a Fraction	Rational or Irrational?
1.75	7/4	Rational
.001	1/1000	Rational
$\sqrt{2}$ (square root of 2)	?	Irrational !

Square Root of 2

Let's look at the square root of 2 more closely.



When we draw a square of size "1", what is the distance across the diagonal?

The answer is the **square root of 2**, which is **1.4142135623730950...(etc)**

But it is not a number like 3, or five-thirds, or anything like that ...

in fact we **cannot** write the square root of 2 using a ratio of two numbers

I explain **why** Is It Irrational?

It can't be written in ratio

... and so we know it is **an irrational number**

Some examples of irrational numbers

Pi is a famous irrational number. People have calculated Pi to over a quadrillion decimal places and still there is no pattern. The first few digits look like this:

3.1415926535897932384626433832795 (and more ...)

Many square roots, cube roots, etc are also irrational numbers. Examples:

√3 1.7320508075688772935274463415059 (etc)

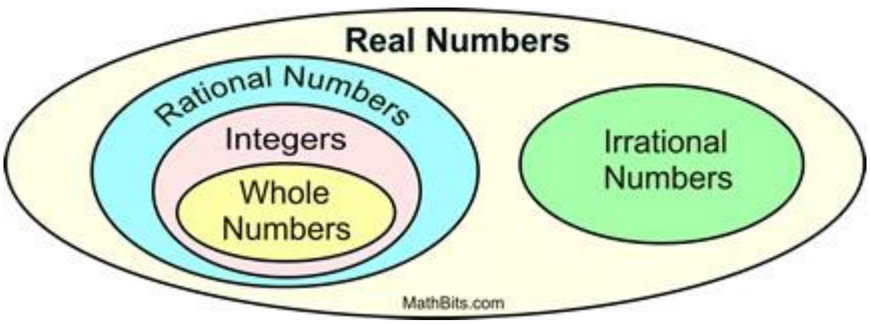
√99 9.9498743710661995473447982100121 (etc)

But √4 = 2 (rational), and √9 = 3 (rational) ...... so **not all** roots are irrational.

- $\boldsymbol{\pi} \times \boldsymbol{\pi} = \boldsymbol{\pi}^2$ is irrational
- But **√2 × √2** = **2** is **rational**

multiplying irrational numbers **might** result in a rational number!

This diagram shows clear points about rational and irrational numbers



When a rational number fraction is divided to form a decimal value,
it becomes a **terminating** or **repeating decimal**.

Non-Terminating, Non-Repeating Decimal

Definition

A non-terminating, non-repeating decimal is a decimal number that continues endlessly, with no group of digits repeating endlessly.

Decimals of this type cannot be represented as fractions, and as a result are irrational numbers.

Examples

Pi is a non-terminating, non-repeating decimal. π = 3.141 592 653 589 793 238 462 643 383 279 ...

e is a non-terminating, non-repeating decimal. e = 2.718 281 828 459 045 235 360 287 471 352 ...

Non-terminating, non-repeating decimals can be easily created by using a pattern. Some examples are listed below:

- 0.10100100010000100000100000010000000 ...
- 28.1211221112221111222211111222222111111222222

$$\frac{3}{4} \text{ can be represented as } 4\overline{)3.00}^{\,0.75} \text{ which is a terminating decimal.}$$

$$\frac{2}{3} \text{ can be represented as } 3\overline{)2.0}^{\,0.\overline{6}} \text{ which is a repeating decimal.}$$

$$\frac{2}{3} = \ 0.666666\ldots\ldots\ldots \text{ which can be written as } 0.\overline{6}$$

Similarly

$\frac{1}{3} = 0.33333\ldots\ldots\ldots = 0.\overline{3}$

$\frac{5}{9} = 0.55555\ldots\ldots\ldots. = 0.\overline{5}$

$\frac{14}{11} = 1.27272727\ldots.. = 1.\overline{27}$

Example: 0.5, 2.456, 123.456, etc. are all examples of **terminating decimals**. **Non terminating decimals** are those which keep on continuing after **decimal** point (i.e. they go on forever). They don't come to end or if they do it is after a long interval.

To convert a repeating decimal to a fraction:

Q1. Show that $0.\overline{6}$ can be expressed in the form p/q, Where p, q are integers ands $q \neq 0$.

Let us assume that x=0.6= 0.6666...........------> (1)

Now, we need to Multiply equation (1) with 10

10x = 10 × (0.666...)

10x = 6.666....... →(2)

Subtracting equation(2) from equation(1)

10 x- x = 6.6666.... - 0.6666...

9x = 6

x= 9/6 = 2/3

Q2. $0.4\overline{7}$ = 0.4777777----> (1)

Now we need to multiply with equation (1) with 10

10x = 10 × (0.47777)...

10x= 4.7777....→(2)

Now we need to subtract equation (2) from equation (1)

10x-x = 4.7777...- 0.47777.....

9x = 4.3000

x= 4.3 /9

Therefore x = 43/90

OR

10x = 4.77777…..→(2)

100x = 10x (4.7777…)

100x = 47.777…… →(3)

Now we need to subtract equation (2) from equation (3)

100x – 10x = 43

90x = 43

X = 43/90

Q3. $0.\overline{001}$

Let us assume that x= 0.001 = 0.001001......(1)

Now let us multiply equation with 1000

1000 x = 1000 × (0.001001...)

1000x = 001.001001………..(2)

Now we need to subtract (1) from equation (1)

$1000x - x = 1.001001... - 0.001001...$

$999x = 1$

$x = 1/999$