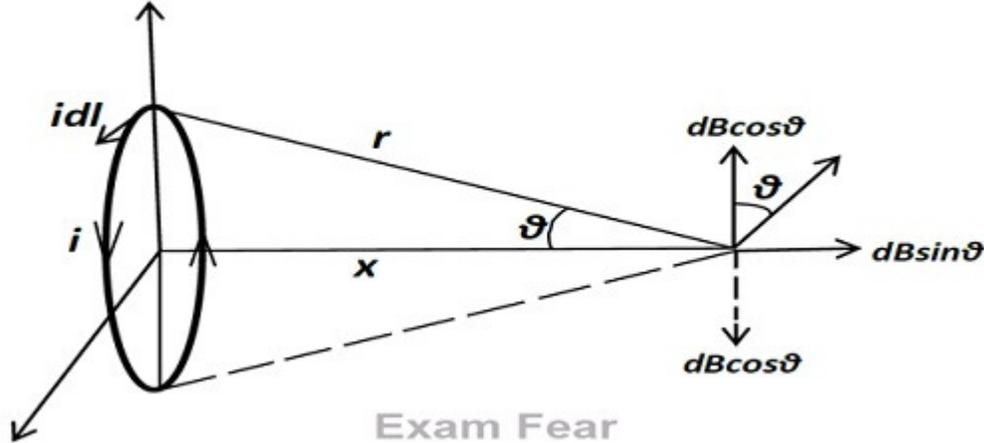


MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT LOOP:



Magnetic field $d\mathbf{B}$ at point P due to current element idl , making right angle to the line joining point P and current element, will be given by Biot-Savart law as:

$$d\mathbf{B} = (\mu_0/4\pi)idl \sin(90^\circ)/r^2 = (\mu_0/4\pi)idl/r^2$$

- As we can see in the diagram, the magnetic field $d\mathbf{B}$ will have 2 component, i) the vertical component $d\mathbf{B}\cos\theta$, and ii) the horizontal component $d\mathbf{B}\sin\theta$
- It is also evident from the diagram that the vertical component $d\mathbf{B}\cos\theta$ will be cancelled by the equal and opposite component due to current element at the opposite of the above current element (due to symmetry).
- So, the total magnetic field will only be due to the horizontal component ($d\mathbf{B}\sin\theta$) along the positive x -axis

$$\underline{d\mathbf{B}\sin\theta = (\mu_0/4\pi)idl(\sin\theta)/r^2}$$

$$\underline{\sin\theta = R/r = R/\sqrt{(x^2 + R^2)}}$$

$$\underline{\therefore d\mathbf{B}\sin\theta = (\mu_0/4\pi)iRdl/(x^2 + R^2)^{3/2}}$$

- So, the total magnetic field will be:

$$dB \sin \theta = \frac{\mu_o}{4\pi} \frac{iRdl}{(x^2 + R^2)^{\frac{3}{2}}}$$

$$B_{total} = \int dB \sin \theta = \frac{\mu_o}{4\pi} \frac{iR}{(x^2 + R^2)^{\frac{3}{2}}} \int_0^{2\pi R} dl$$

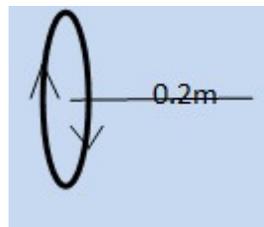
$$\therefore B_{total} = \frac{\mu_o}{4\pi} \frac{iR}{(x^2 + R^2)^{\frac{3}{2}}} 2\pi R = \frac{\mu_o i R^2}{2(x^2 + R^2)^{\frac{3}{2}}}$$

o For magnetic field at the center of current loop ($x = 0$):

$$B_{total} = \frac{\mu_o i R^2}{2(0^2 + R^2)^{\frac{3}{2}}} = \frac{\mu_o i R^2}{2R^3} = \boxed{\frac{\mu_o i}{2R}}$$

Numerical Problem:

1) A circular coil of wire has 100 turns of radius 8cm, and carrying a current of 0.4A in clockwise direction when viewed from the right side. Find the magnitude and direction of magnetic field: i) at the center of coil, and ii) at a distance of 20cm from the center of coil towards the right and normal to the coil.



Solution: Given, $N = 100$, $r = 0.08\text{m}$, $i = 0.4\text{A}$, $x = 0.2\text{m}$

1. i) Magnetic field at the center of circular coil is given by:

$$B = \mu_o Ni / (2r)$$

$$B = 4\pi \times 10^{-7} \times 100 \times 0.4 / (2 \times 0.08) = \underline{\underline{3.14 \times 10^{-4} \text{T} = 3.14 \text{G (ans)}}$$

Direction of magnetic field will be normal to the plane of coil, and from right to left side of coil.

1. ii) Magnetic field at an axial distance from center of coil is given by:

$$B = \mu_o Ni R^2 / (x^2 + R^2)^{(3/2)}$$

$$B = 4\pi \times 10^{-7} \times 100 \times 0.4 \times 0.0064 / (0.0064 + 0.0400)^{(3/2)} = 3.22 \times 10^{-7} / 0.00999$$

$$\therefore B = \underline{\underline{3.22 \times 10^{-5} \text{T} = 0.322 \text{G (ans)}}$$