

AMPERE'S CIRCUITAL LAW:

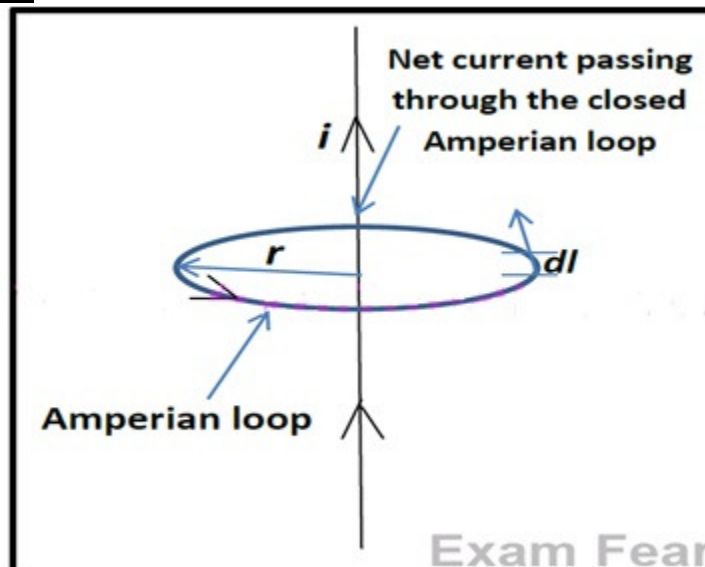
- Ampere's circuital law states that line integral of magnetic field forming a closed loop around the current(i) carrying wire, in the plane normal to the current, is equal to the μ_0 times the net current passing through the close loop.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

Here $\mu_0 = \text{permeability of free space} = 4\pi \times 10^{-7} \text{ N/A}^2$

- This law is based on the assumption that the closed loop consists of small elemental parts of length dl , and the total magnetic field of the closed loop will be the integral of magnetic field and the length of these elements $\oint \vec{B} \cdot d\vec{l}$. This closed loop is called Amperian loop
- Further, this integral will be equal to the multiplication of net current passing through this closed loop and the permeability of free space($\mu_0 i$)

Proof-1(Regular coil):



To prove: $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$

Starting from the left hand side, we can see in the diagram that angle between the element dl and magnetic field B is 0°

$$\int \vec{B} \cdot d\vec{l} = \int B dl \cos 0 = B \int dl$$

We know that magnetic field due to a long current carrying wire is:

$$B = \mu_0 i / (2\pi r)$$

Also, the integral of element will form the whole circle of circumference ($2\pi r$):

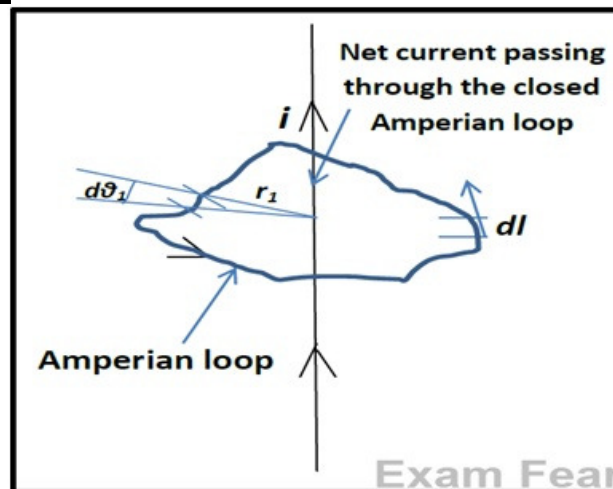
$$\int dl = 2\pi r$$

Now putting the value of B and $\int dl$ in the equation, we get:

$$B \int dl = \mu_0 i / (2\pi r) \times 2\pi r = \mu_0 i$$

$$\therefore \int B \cdot dl = \mu_0 i$$

Proof-2 (Irregular coil):



To prove: $\int B \cdot dl = \mu_0 i$

Starting from the left hand side:

$$\int B \cdot dl_1 = \int \mu_0 i / (2\pi r_1) \times dl_1$$

We know that: $d\theta_1 = dl_1 / r_1$

$$\therefore \int \mu_0 i / (2\pi r_1) \times dl_1 = \mu_0 i / (2\pi) \int d\theta_1 = \mu_0 i$$

$$\int B \cdot dl = \mu_0 i$$

Note:

- The above two derivations proves that magnetic field at a point doesn't depend on the shape of the Amperian loop.
- Magnetic field is same at every point in the Amperian loop (magnetic field possesses cylindrical symmetry)
- Direction of magnetic field at any point on the Amperian loop is tangential to the circle formed at that point with wire passing through the center, and the direction could be calculated by right hand thumb rule where, on holding the current carrying wire such that the extended thumb shows the direction of current in the wire, then the curling of rest of the 4 fingers represent the direction of rotation of magnetic field.