3.3.2 Logical Operators

TRUTH TABLE

A *Truth Table* is a table which represents all the possible values of logical variables / statements along with all the possible results of the given combinations of values.

Before we start discussion about logical operators, let us first understand *what a Truth Table is.*

For example, following logical statements can have only one of the *two* values (TRUE (YES) or FALSE (NO))

1. I want to have tea. 2. Tea is readily available.

Let us represent all the possible combinations of values these statements can have in the tabular form :

I want to have tea	T	T	F	F
Tea is readily available	T	F	T	F
(Result) I'll have tea	T	F	F	F

T represents True
F represents False

Or If we represent first statement as *X* and second statement as *Y* and result as *R* then the above table can also be written as follows :

Table 3.1

X	Y	R
1	1	1
1	0	0
0	1	0
0	0	0

1 represents TRUE value and
0 represents FALSE value

TAUTOLOGY

If the result of any logical statement or expression is always TRUE or 1 for all input combinations, it is called Tautology.

This is a truth table *i.e., table of truth values of truth functions.*

Now let us proceed with our discussion about logical operators *i.e.,*

- NOT Operator
- OR Operator
- AND Operator

FALLACY

If the result of any logical statement or expression is always FALSE or 0 for all inpu combinations, it is called Fallacy

NOT Operator

This operator **operates on single variable** and operation performed by NOT operator is calle *complementation* and the symbol we use for it is ¯ (bar). Thus $\overline{X}$ means complement of X and $\overline{Y}$ means complement of YZ. As we know, the variables used in boolean equations have a uniq characteristic that they may assume only one of two possible values 0 and 1, where 0 deno FALSE and 1 denotes TRUE value. Thus the complement operation can be defined quite simp

$$\overline{0} = 1$$
$$\overline{1} = 0$$

Table 3.2 *Truth Table for NOT Operators*

X	$\overline{X}$ (i.e., NOT X)
0	1
1	0

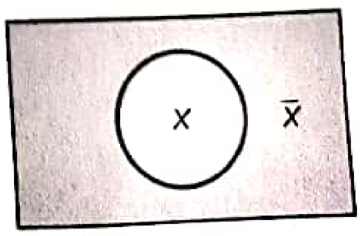


Figure 3.1 Venn diagram for

Several other symbols *e.g.* ~, ... are ... read ... *X'* is read ... ~ X is read is 'negation of X' and if ' is used then X' is read ... singular or unary operation as it operates on single variable. Venn diagram for X is given in Fig. 3.1 where shaded area depicts $\bar{X}$.

OR Operator

A second important operator in boolean algebra is OR operator which denotes operation called *logical addition* and the symbol we use for it is +. The + symbol, therefore, does not have the 'normal' meaning, but is a *logical addition* or logical OR symbol. Thus X + Y can be read as **X OR Y**. For OR operation the possible input and output combinations are as follows :

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 1$$

Shaded Portion shows X + Y

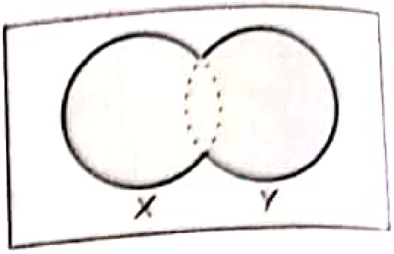


Figure 3.2 Venn diagram for X + Y.

And the truth table of OR operator is given below :

Table 3.3 *Truth Table for OR Operator*

X	Y	X + Y (i.e., X OR Y)
0	0	0
0	1	1
1	0	1
1	1	1

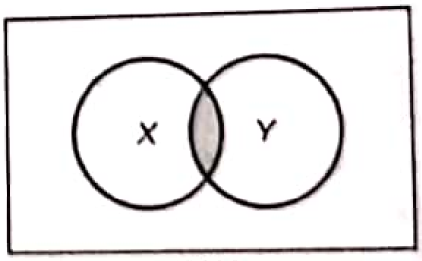
Note that when any one of X and Y is 1, X + Y is 1.

To avoid ambiguity, there are other symbols *e.g.*, *U*, *v*, and *V* have been recommended as replacements for the + sign. Computer people still use the + sign, however, which was the symbol originally proposed by Boole. Venn diagram for X + Y is given (Fig. 3.2), where shaded area depicts X + Y.

AND Operator

AND operator performs another important operation of boolean algebra called *logical multiplication* and the symbol for AND operation is (.) dot. Thus X . Y will be read as X AND Y. The rules for AND operation are :

$$0 . 0 = 0$$
$$0 . 1 = 0$$
$$1 . 0 = 0$$
$$1 . 1 = 1$$



Shaded Portion shows X · Y

X.Y

Figure 3.3 Venn diagram for (X . Y).

and the truth table for AND is as follows :

Table 3.4 *Truth Table for AND Operator*

X	Y	X . Y (i.e., X AND Y)
0	0	0
0	1	0
1	0	0
1	1	1

Note that only when both X and Y are 1's, then XY has the result 1.

Venn diagram for X . Y is given in Fig. 3.3, where shaded area depicts (X . Y).